SHOULD WE BELIEVE ATMOSPHERIC TEMPERATURES MEASURED BY ENTRY ACCELEROMETERS TRAVELLING AT “SLOW” NEAR-SONIC SPEEDS?

Paul Withers$^{(1,2)}$

$^{(1)}$*Center for Space Physics, Boston University, 725 Commonwealth Avenue, Boston, MA 02215, USA*
(email: withers@bu.edu)
$^{(2)}$*Planetary and Space Science Research Institute, Open University, Walton Hall, Milton Keynes, MK7 6AA, Great Britain*

ABSTRACT

Mars Pathfinder’s Accelerometer instrument measured an unexpected and large temperature inversion between 10 and 20 km altitude. Other instruments have failed to detect similar temperature inversions. I test whether this inversion is real or not by examining what changes have to be made to the assumptions in the accelerometer data processing to obtain a more “expected” temperature profile. Changes in derived temperature of up to 30K, or 15%, are necessary, which correspond to changes in derived density of up to 25% and changes in derived pressure of up to 10%. If the drag coefficient is changed to satisfy this, then instead of decreasing from 1.6 to 1.4 from 20 km to 10 km, the drag coefficient must increase from 1.6 to 1.8 instead. If winds are invoked, then speeds of 60 m s^{-1} are necessary, four times greater than those predicted. Refinements to the equation of hydrostatic equilibrium modify the temperature profile by an order of magnitude less than the desired amount. Unrealistically large instrument drifts of 0.5 – 1.0 m s^{-2} are needed to adjust the temperature profile as desired. However, rotational contributions to the accelerations may have the necessary magnitude and direction to make this correction. Determining whether this hypothesis is true will require further study of the rigid body equations of motion, with detailed knowledge of the positions of all six accelerometers.

The paradox concerning this inversion is not yet resolved. It is important to resolve it because the paradox has some startling implications. At one extreme, are temperature profiles derived from accelerometers inherently inaccurate by 20 K or more? At the other extreme, are RS temperature profiles inaccurate by this same amount?

1 INTRODUCTION

The aim of this paper is to investigate an unusual temperature inversion measured by the accelerometer (ACC) instrument on Mars Pathfinder (MPF) during its descent through the atmosphere of Mars. A temperature inversion occurs when temperature increases as altitude increases within an atmosphere. Temperatures usually decrease as altitude increases. Neither models nor other observations have shown similar inversions. Is the inversion seen by MPF real? I will attempt to remove this temperature inversion from the MPF data by testing the limits of the assumptions underlying the data processing. If I suceed, then the temperature inversion may be an artifact of the data acquisition and processing. If I do not succeed, then the temperature inversion is robust and real, and its presence in this dataset and absence from other datasets must be explained.

Fig. 1 shows the vertical temperature profile derived from MPF’s ACC data using the techniques of Withers et al. [10] and Withers [11]. It is very similar, but not identical, to the profiles derived by other workers [6, 9]. Uncertainties are not shown because my data processing software has not yet been extended to include a thorough error analysis. Errors are shown in Magalhães et al. [6]. The MPF ACC Science Team has archived its reconstructed trajectory and atmospheric structure, with extensive documentation, at the Planetary Data System (PDS) [8]. A noteworthy feature of this temperature profiles is the large temperature inversion below 20 km. Some workers have suggested that it is due to radiative cooling from a water ice cloud [4, 1, 2]. However, repeated Mars Global Surveyor (MGS) Radio Science (RS) occultation measurements of temperature and pressure in the lower atmosphere at similar latitudes, longitudes, and local solar times (LSTs) one Mars year later do not reveal any trace of an inversion [5]. Hinson and Wilson [5] do see inversions at other latitudes and longitudes at this LST and season (Ls),

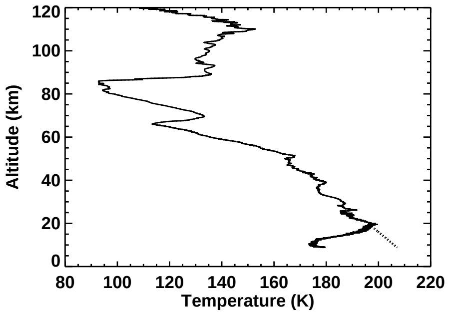

Figure 1: Temperature as a function of altitude for MPF (dashed line) and a model (solid line). The vertical axis has a linear scale from 0 to 50 km. Figure from Colaprete and Toon [2].

but they state that the RS inversions have distinctly different structures to the MPF inversion. Based on visual inspection of some RS inversions shown in Fig. 2 of Hinson and Wilson [5], I think that the RS inversions are narrower in width than the MPF inversion and have an asymmetric vertical profile, whereas the MPF inversion has a more symmetric profile. Colaprete and Toon [2] show (their Fig. 5, reproduced in Fig. 2) a qualitative comparison between the MPF inversion and a model inversion. A quantitative comparison is not discussed. The model inversion appears to have the same discrepancies with respect to the MPF inversion as the observed RS inversions in Hinson and Wilson [5]. Comparing static stabilities might be the best way to quantify the differences between RS inversions, the MPF inversion, and model inversions [6, 5].

2 TEMPERATURE PROFILES

Fig. 3, courtesy of John Wilson, shows an MGS Thermal Emission Spectrometer (TES) profile, which has poor vertical resolution, several MGS RS profiles, which have good vertical resolution, and the MPF ACC profile, which also has good vertical resolution. Nothing like the MPF inversion is seen in the TES or RS data. The MPF temperature maximum occurs near 20 km. The MPF temperature inversion can be removed, and the profile made more consistent with the TES and RS data, if temperatures, T, below 20 km altitude are replaced by the following equation:

$$T/K = 216 - z/km \tag{1}$$

where z is altitude. Throughout this paper, "altitude" means radial distance above the MPF landing site, which 3389.715 km from the centre of mass of

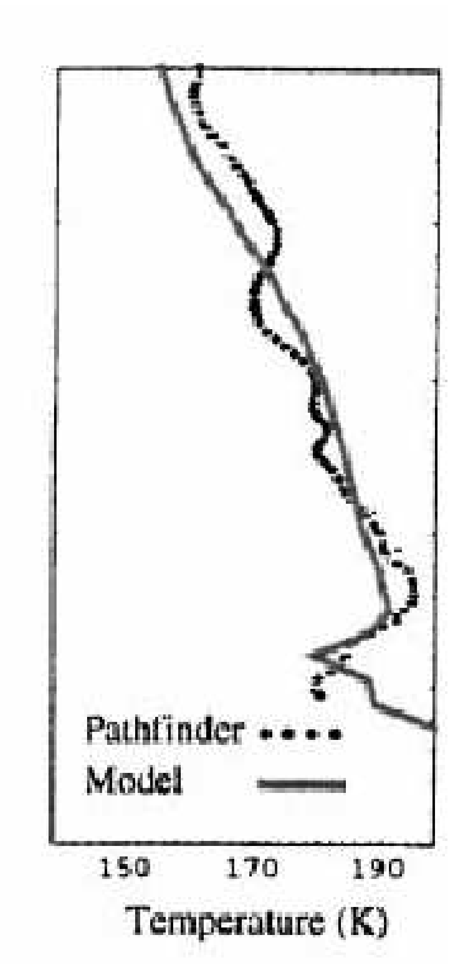

Figure 2: Original temperature profile from MPF (solid line) and desired temperature profile (dotted line).

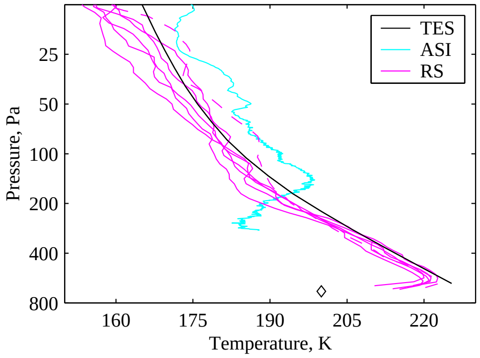

Figure 3: One TES temperature profile (smooth line), several RS profiles (extending to the base of the figure), and the MPF ACC temperature profile (with large temperature inversion). Figure provided by John Wilson.

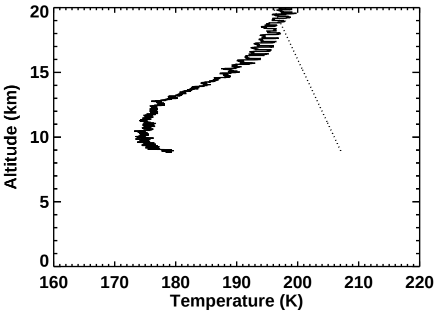

Figure 4: Original temperature profile from MPF (solid line) and desired temperature profile (dotted line).

Mars. This "desired" temperature profile is also plotted on Fig. 1. Fig 4 shows the original and desired temperatures profiles below 20 km altitude. A 30 K shift in temperature will not be easy to obtain.

3 DENSITY AND PRESSURE

Density, ρ, pressure, p, and temperature are related by the ideal gas law:

$$T = \frac{m_{mean}}{k_{Boltzmann}} \times \frac{p}{\rho} \tag{2}$$

where m_{mean} is the mean mass of an atmospheric molecule and $k_{Boltzmann}$ is Boltzmann's constant. m_{mean} may vary slightly in the lower atmosphere of Mars, but not by enough to change the temperature profile as desired. Hence atmospheric density or pressure or both must be altered in Eqn. 2 to alter the temperature. Atmospheric pressure is related to atmospheric density by the equation of hydrostatic equilibrium, Eqn. 3.

$$p(z) = p(z_0) - \int_{z_0}^{z} \rho g dz \tag{3}$$

where g is the magnitude of the acceleration due to gravity. How must density, and pressure change to be consistent with this desired temperature profile? Since pressure is effectively vertically-integrated density, its value just below 20 km is strongly influenced by the densities above 20 km. So density just below 20 km must decrease from its original value to shift the temperature from its original to its desired value. As we progress downwards, density must continue to decrease from its original value. Pressures must decrease also, since they are vertically-integrated densities, but not by as much. Fractional changes in density and pressure required to give the desired temperature profile are shown in Fig. 5. The fractional change in density cannot stabilize, because then the fractional change in pressure would also stabilize (at a lower altitude) and the temperature would be back to its original value. This is quite a strong constraint, which also applies to anything else that affects the density profile.

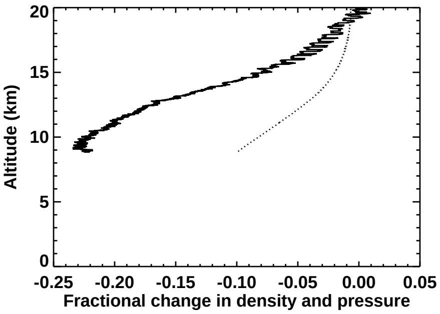

Figure 5: Fractional change in density (solid line) and pressure (dotted line) necessary to obtain desired temperature profile.

The MPF ACC measured accelerations and then converted them into densities. Pressures and temperatures were derived from the density profile using Eqns. 3 and 2. Accelerations were converted into densities using the drag equation:

$$\rho = \frac{-2m}{C_A A} \times \frac{a_z}{v_R^2} \tag{4}$$

where m is the mass of the spacecraft, A is the reference area of the spacecraft, C_A is the axial force coefficient appropriate to the present angle of attack, atmospheric composition, density, and temperature, a_z is the acceleration along the spacecraft's z-axis, and v_R is the speed of the spacecraft relative to the atmosphere. I used aerodynamic databases from Moss et al. [7] and Gnoffo et al. [3], generally assuming an angle of attack of zero.

If the desired changes in ρ were due solely to changes in m, m would have to decrease by 20%, or over 100 kg. This is unrealistic. If the desired changes in ρ were due solely to changes in A, the spacecraft radius would have to increase by 10%, or 13 cm. This is also unrealistic. Even if the magnitude of the changes were reasonable, it would still be challenging to get the desired change as a function of altitude and to explain why the change starts below 20 km.

4 AERODYNAMICS

If the desired changes in ρ were due solely to changes in C_A, then the vertical profile of C_A, shown in Fig. 6, would change dramatically. Is this realistic? The MPF aerodynamic database was generated

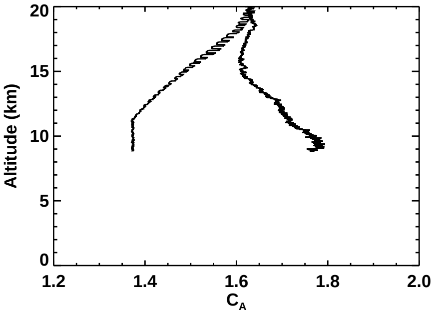

Figure 6: Original vertical profile of C_A for MPF reconstruction (values 1.4 – 1.6) and profile of C_A required to obtain desired temperature profile (values 1.6 – 1.8).

by running computationally-intensive numerical simultations under a discrete set of flow conditions. One value of C_A was extracted from each simulation, yet MPF's actual trajectory encompassed an infinite continuum of flow conditions. Appropriate values of C_A are formed by interpolation within the finite set of simulations. The interpolation might not be accurate and the simulations themselves might not be accurate. It has been stated that uncertainties in C_A for MPF's atmospheric entry are from 1 to 3% [6]. 5 to 10% seems more realistic to me, with greatest uncertainties at low altitudes. Below 20 km altitude, flow conditions for which there are numerical simulations are at 18 km and at 9 km. The shape of the large decrease in C_A between these altitudes is unconstrained by simulations or by observational data. This will make interpolation of C_A in this region very difficult. It does not matter much if the simulated value of C_A is wrong when C_A is not changing much. Derived temperatures are quite insensitive to this. However, when C_A is changing rapidly, derived temperatures are much more sensitive. Even allowing for reasonable errors in simulated values of C_A and in interpolating between simulations, the changes in C_A necessary to obtain the desired temperature profile seem unreasonably large.

Suppose that the simulations of Gnoffo et al. [3] at Mach number (Ma) 1.9 and 2.0 are somehow systematically flawed and they should have generated $C_A \approx 1.5$ instead of the actual values of 1.3. Rederiving the temperature profile with this updated aerodynamic database *still* produces a large temperature inversion, Fig. 7. It appears that C_A must increase by 15% as altitude decreases from 20 km to 8 km to obtain the desired temperature profile, though the numerical simulations predict that it decreases by 15%.

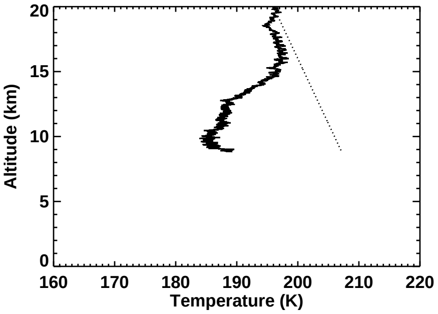

Figure 7: Temperature profile from MPF, assuming that values of $C_A = 1.3$ in the aerodynamic database should actually be 1.5, (solid line) and desired temperature profile (dotted line).

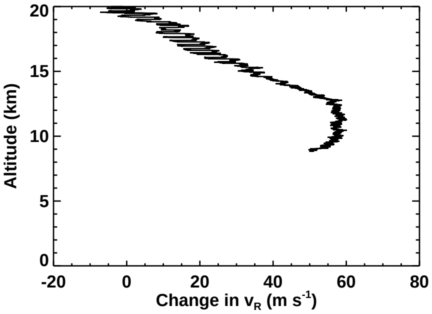

Figure 8: Change in v_R necessary to obtain the desired temperature profile.

5 WIND

Atmospheric dynamics can affect the results of Eqn. 4. Any wind in the atmosphere affects v_R. Fig. 8 shows the difference between the magnitude of v_R necessary to obtain the desired temperature profile and the magnitude of v_R in the original reconstruction. The horizontal wind necessary to account for this change, assuming that it is blowing in the most favourable direction, is 10 – 20% greater than this difference. These wind speeds are significantly greater than the 15 m s^{-1} speeds predicted by general circulation models for these conditions [6].

We can combine considerations of winds and errors in C_A. Suppose $C_A \approx 1.5$ at Ma=1.9 and Ma=2.0 instead of the actual 1.3 — what horizontal winds are needed in this case to obtain the desired temperature profile? Fig. 9 shows the difference in magnitudes of v_R for this case. The change to C_A is about twice as much as is reasonable and the derived wind speed is about twice as much as is reasonable as well — even if it is blowing in the most favourable direction. These techniques have not been successful at removing the

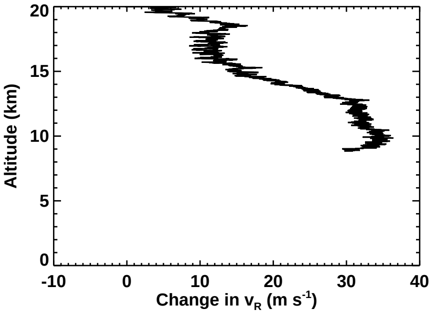

Figure 9: Change in v_R necessary to obtain the desired temperature profile, assuming that values of C_A = 1.3 in the aerodynamic database should actually be 1.5.

temperature inversion.

6 REFINED HYDROSTATIC EQUILIBRIUM

The equation of hydrostatic equilibrium (Eqn. 3) is not completely accurate. It has neglected higher order gravitational terms, terms due to planetary rotation, and the effects of atmospheric dynamics. Correcting for the first two categories gives:

$$\Delta p = \rho \left(g_{eff,r} \Delta r + g_{eff,\theta} r \Delta \theta + g_{eff,\phi} r sin\theta \Delta \phi \right) \tag{5}$$

$$g_{eff,r} = \frac{-GM}{r^2} - \cdots \tag{6}$$

$$\frac{9}{2} \frac{GM}{r^2} \left(\frac{r_{ref}}{r} \right)^2 \left(\cos\theta \cos\theta - 1 \right) C_{20} + \cdots \tag{7}$$

$$r \Omega^2 \sin\theta \sin\theta \tag{8}$$

$$g_{eff,\theta} = \frac{-3GM}{r^2} \left(\frac{r_{ref}}{r} \right)^2 \sqrt{5} \sin\theta \cos\theta C_{20} + \cdots \tag{9}$$

$$r \Omega^2 \sin\theta \cos\theta \tag{10}$$

$$g_{eff,\phi} = 0 \tag{11}$$

where the terms have their usual meanings. A latitude of 20° corresponds to $\sin\theta \cos\theta = 0.3$. $C_{20} \sim -10^{-3}$, $GM/r^2 = 3.7$ m s^{-2}, and $r\Omega^2 = 0.01$ m s^{-2}. The effect of these corrections is critically dependent on the trajectory. A shallow entry trajectory along a meridian of constant longitude will have a large correction. A steep entry trajectory along a circle of constant latitude will have a small correction. I have rederived pressure and temperature profiles for MPF using this correction; both change by less then 1% at all altitudes. Further corrections to hydrostatic equilibrium due to atmospheric dynamics are about an order of magnitude smaller than these corrections.

Horizontal gradients in the atmosphere, regardless of their cause, can be investigated. If $\partial \ln \rho / \partial \theta = k$ and $\partial \ln \rho / \partial r = H$, if MPF's atmospheric trajectory can be modelled as $r + \alpha \theta =$ constant, and if the atmosphere is isothermal with temperature T_{iso}, then temperatures derived from entry accelerometer data using Eqn. 3 will be given by $T = T_{iso}(1 - kH/\alpha)$. For MPF, $\alpha \sim 40$ km deg^{-1} and $H \sim 10$ km. A reasonable value of k is 0.01 deg^{-1}, which alters temperatures by a few tenths of one percent.

7 ANGLE OF ATTACK

Eqn. 4 conceals a dependence upon angle of attack, α. α is not measured directly during the entry, it is inferred indirectly from the ratio of axial to normal accelerations using the ratio of axial to normal force coefficients. If force coefficients in the aerodynamic database are incorrect by a few percent, then α could also be inaccurate. Does allowing α to change from its nominal value help obtain the derived temperature profile? Nominal values of α are so close to zero that the only way changes in α can significantly change C_A is for α to increase to some relatively large value. Recall that I need C_A to increase at low altitudes to obtain the desired temperature profile. However, C_A always decreases as α increases. This approach will not help remove the temperature inversion.

8 ANGULAR ACCELERATIONS

Measured accelerations may be corrupted due to rotation of MPF, a rigid body, about its centre of mass. Accelerations along MPF's axis of symmetry are measured by its z-axis accelerometer. This accelerometer is about 5 cm away from the centre of mass along the z-axis. It is much closer to the z-axis than this. Rotational contributions to the measured axial acceleration, such as $\underline{\Omega} \times (\underline{\Omega} \times \underline{r})$ and $\dot{\underline{\Omega}} \times \underline{r}$, might be important. I have not looked at the full rigid body equations of motion to determine how the rotation affects all six (three science and three engineering) accelerometer measurements, but I have made a rough estimate of its importance.

Axial accelerations need to decrease in magnitude by 0.5 – 1.0 m s^{-2} at all altitudes below 20 km to obtain the desired temperature profile. The nominal measurements have magnitudes of 8 m s^{-2} at 9 km and 30 m s^{-2} at 18 km. The instrument resolution at this time is 0.05 m s^{-2}, which is effectively the measurement uncertainty. Such a large instrumental drift, or similar error, can be ruled out. The normal acceleration measurements, on the order of 0.05 m s^{-2}, are so small that they cannot possibly be altered by rotational effects to obtain the desired temperature profile.

$\underline{\Omega}$ was not measured directly by MPF, which did not carry any gyroscopes. MPF had a pre-entry roll about its symmetry axis of 0.06 rad s^{-1}. The time series of measured x-axis and y-axis accelerations show interesting oscillations with an angular frequency $\sim$ 4.5 rad s^{-1} at 20 km and $\sim$ 2.5 rad s^{-1} at 10 km. Overtones are also present. I estimate the maximum angular acceleration contribution to the measured z-axis accelerations to be 4.5 rad s^{-1} $\times$ 4.5 rad s^{-1} $\times$ 50 mm = 1.0 m s^{-2} at 20 km and 2.5 rad s^{-1} $\times$ 2.5 rad s^{-1} $\times$ 50 mm = 0.3 m s^{-2} at 10 km. These are of the desired magnitude. However, until a proper study of the full rigid body equations of motion has been made it would be premature to conclude that this effect is responsible for all of the temperature inversion.

9 CONCLUSIONS

The only effect that seems to offer any hope for removing the temperature inversion from the MPF ACC dataset is due to rotation, possibly assisted by $\sim$ 5% uncertainties in C_A at low Ma. MPF's moments of inertia are known. The positions and orientations of its six accelerometers are presumably also known, though I have not seen them. Measured accelerations can be divided into two contributions: oscillating and non-oscillating. Oscillating contributions are presumably due to rotation, so it might be possible to identify these terms separately within the measured accelerations. Is it possible to deduce MPF's angular velocity vector from these data? Maybe.

Further study of the six-degrees-of-freedom behaviour of MPF during its entry would be helped by analysis of data from Galileo and Pioneer Venus, which measured atmospheric temperatures and pressures directly immediately after parachute deployment, and from MER, which carried 3-axis accelerometers and gyroscopes on both the rover and the backshell.

The paradox concerning this inversion is not yet resolved. It is important to resolve it because the paradox has some startling implications. At one extreme, are temperature profiles derived from accelerometers inherently inaccurate by 20 K or more? At the other extreme, are RS temperature profiles inaccurate by this same amount?

Does this work have any impact on Huygens? I recommend that the Huygens Science Teams investigate how rotation affects each accelerometer instrument onboard and consider how these effects could be mitigated.

More general questions raised by this work concern uncertainties in the aerodynamic database. Should uncertainties in simulated values be represented by a normal distribution, some other distribution, or a possible systematic error? How should uncertainties be assigned to interpolated values of drag coefficients when the way in which the actual drag coefficient changes between simulation points is not known and probably not linear? How do these uncertainties map to uncertainties in derived atmospheric properties? How do they affect derived angles of attack, which in turn affect drag coefficients, and so on?

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