Angle of repose-limited shapes of asteroids

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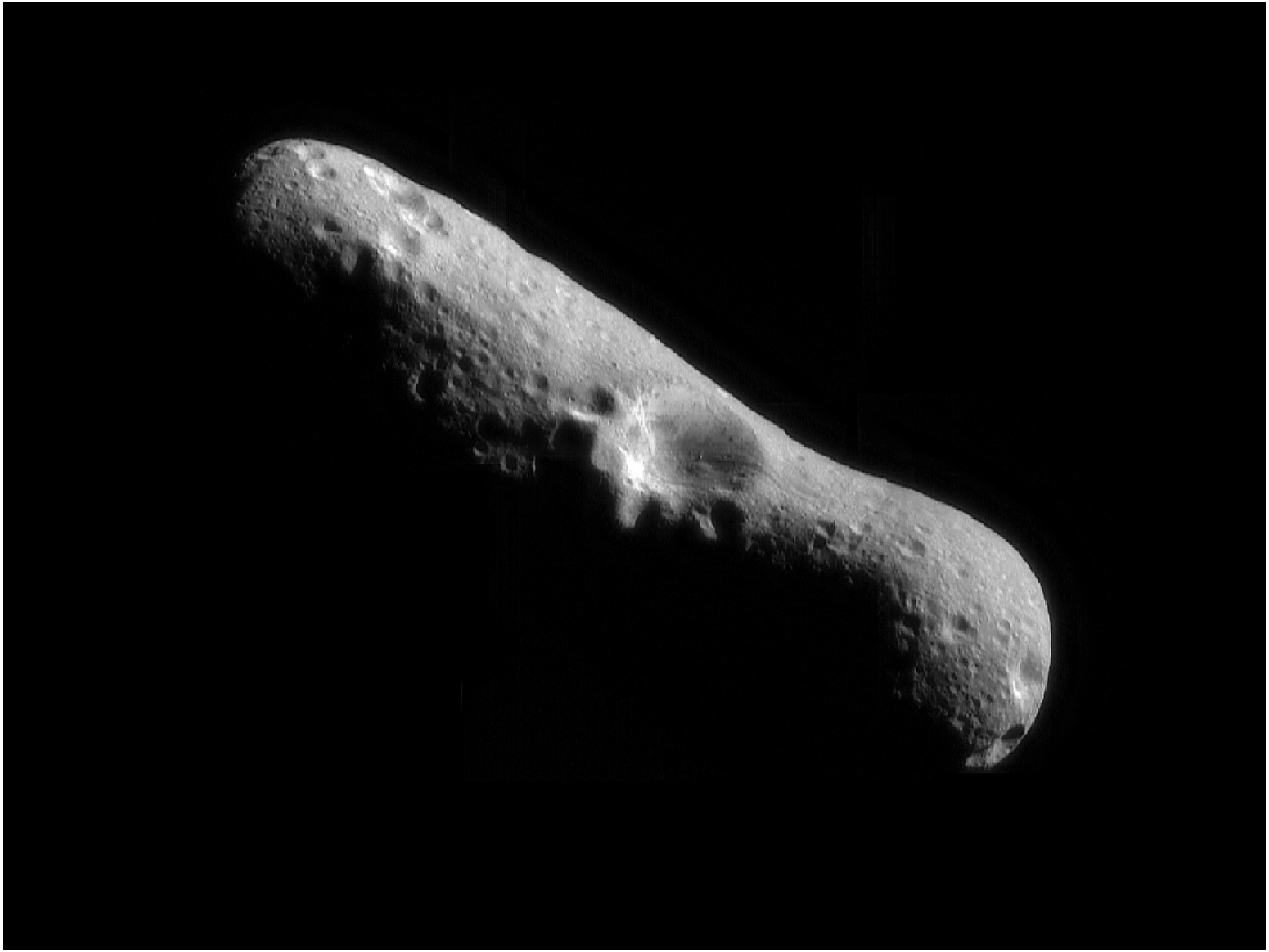
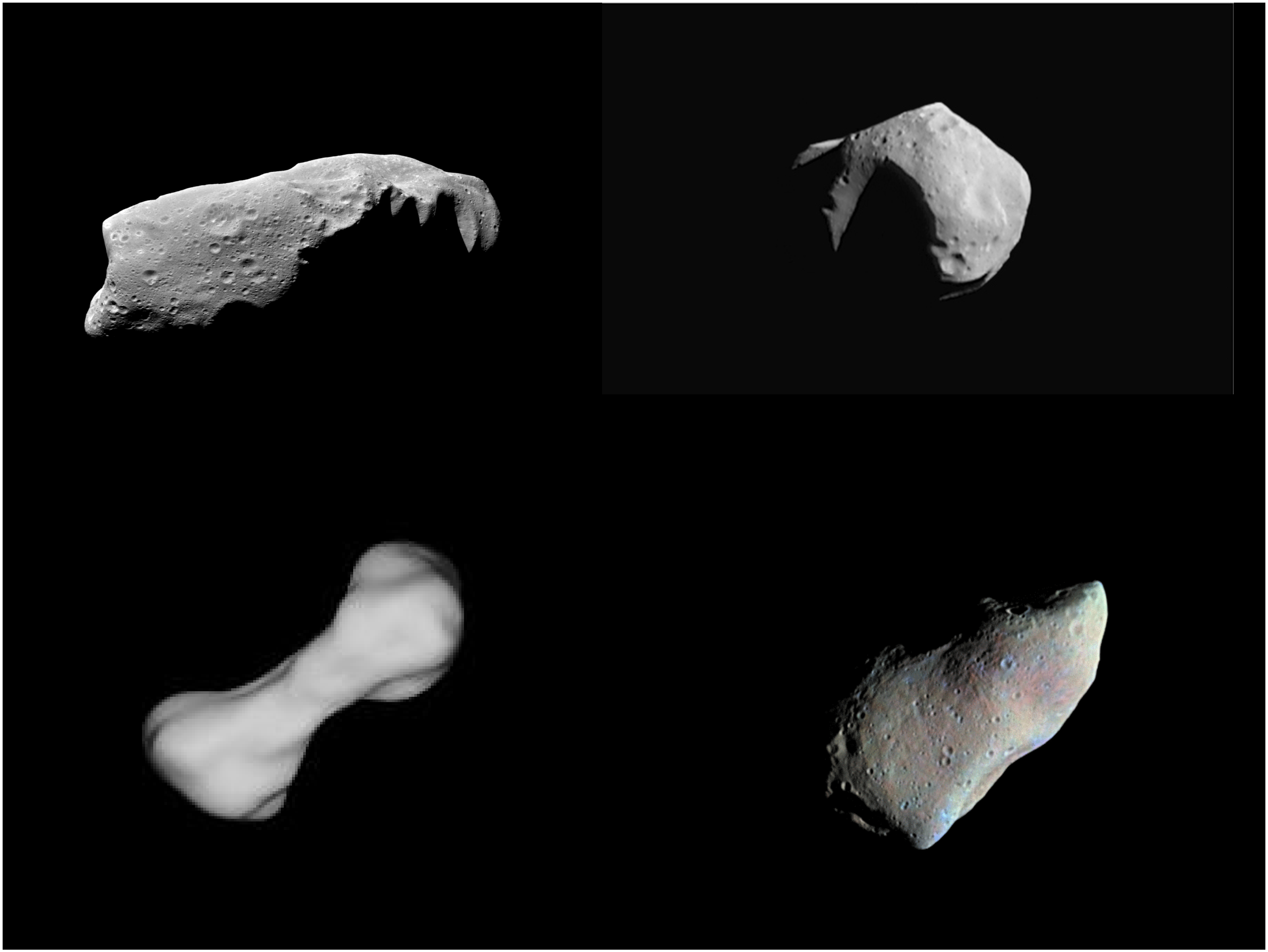
Fall 1999 Surfaces Project
with Jay Melosh

3rd LPLC – 24 May 2000



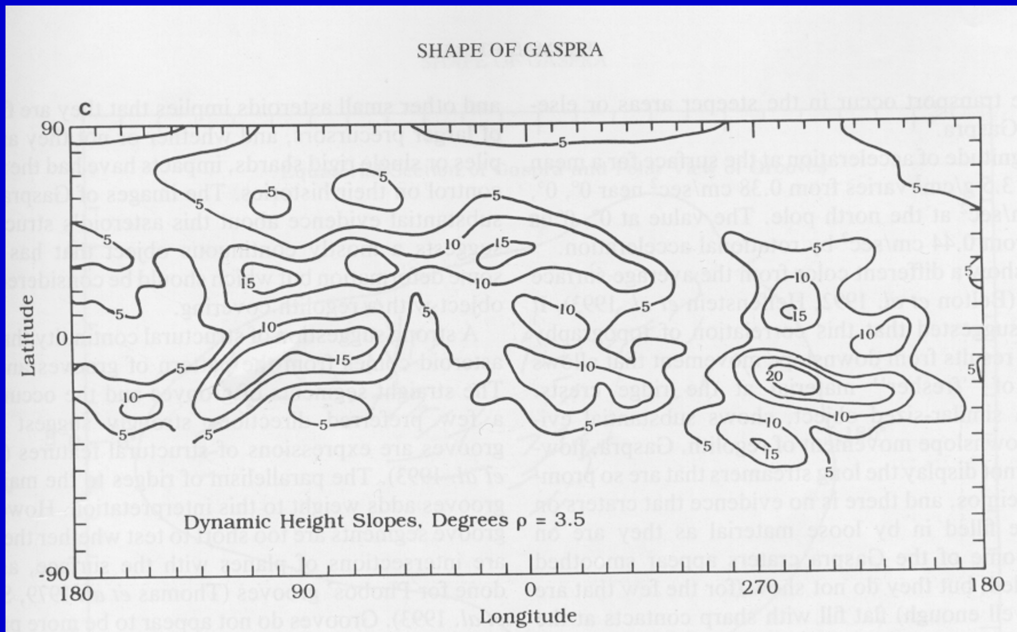
Shapes of rocky bodies

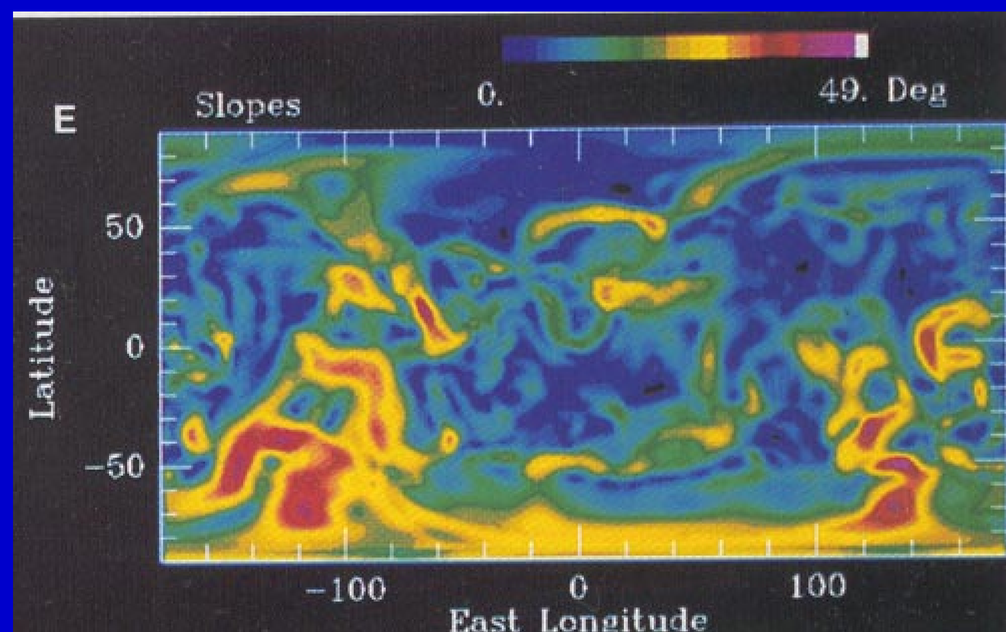
- Large bodies (>300 km) are oblate spheroids with shape controlled by self gravity.
- Small bodies (<300 km) have irregular shapes controlled by material strength.
- Other changes as well.
- Icy bodies have different transition size.



Slopes on Asteroids

- Observed slopes are almost always $< 30^o$, a typical angle of repose.
- Consistent with “rubble pile” model for asteroids, which will support topography via frictional forces.
- Some steep slopes are possible.


SHAPE OF GASPRA
C
Dynamic Height Slopes, Degrees ρ = 3.5
Latitude
90
0
-90
180
90
0
270
180
Longitude


E
Slopes
0.
49. Deg
Latitude
50
0
−50
−100
0
100
East Longitude

Aim of project

- Create shape model for axisymmetric, homogeneous, non-rotating asteroid.
- Asteroid will have no slopes exceeding the angle of repose and as large a mean slope as possible.
- Rotation will be included later.

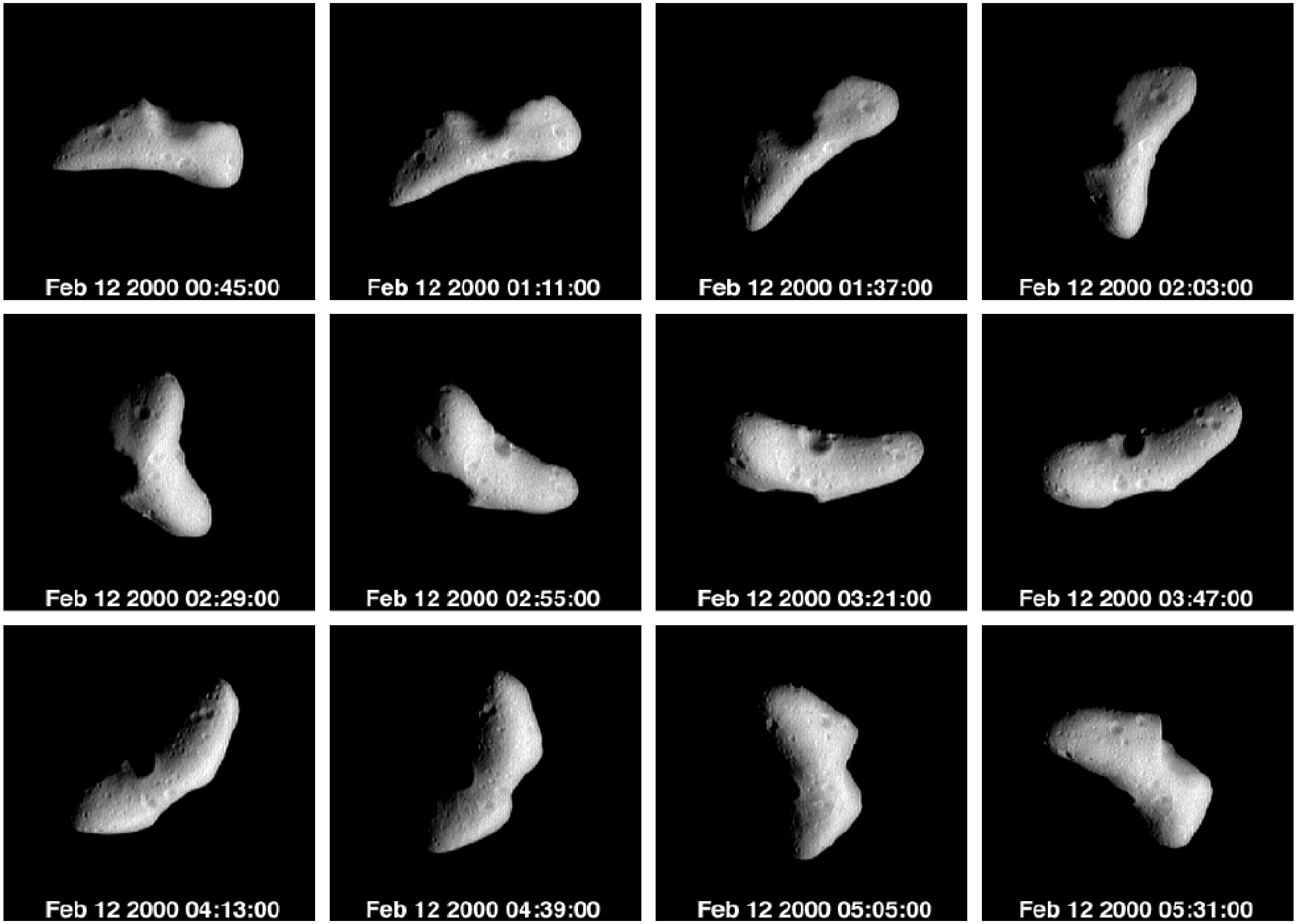
Justification of project

- In the “rubble pile” model, an angle of repose-limited shape is as far removed from a sphere as possible.
- It is an end-member shape whose properties (such as surface roughness and axial ratio) can be compared to observations, testing the “rubble pile” model.

Appproaches

- “Dripping sand” approach
- Iterative approach
- Irregular shape approach
- Ellipsoidal shape approach
- Constrained by timescale of semester project…

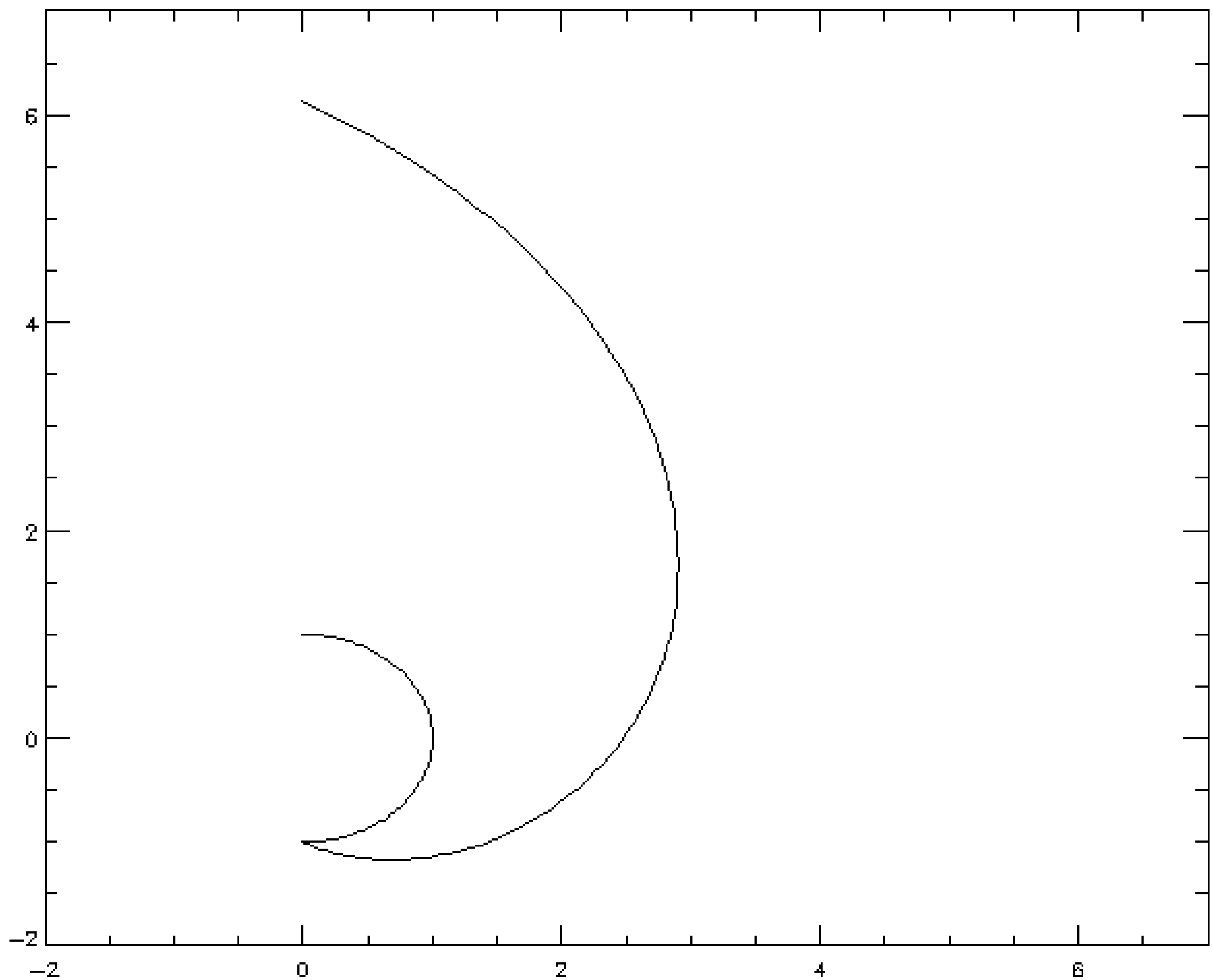
NEAR Approach to Eros



Range = 1800 km

Iterative Approach

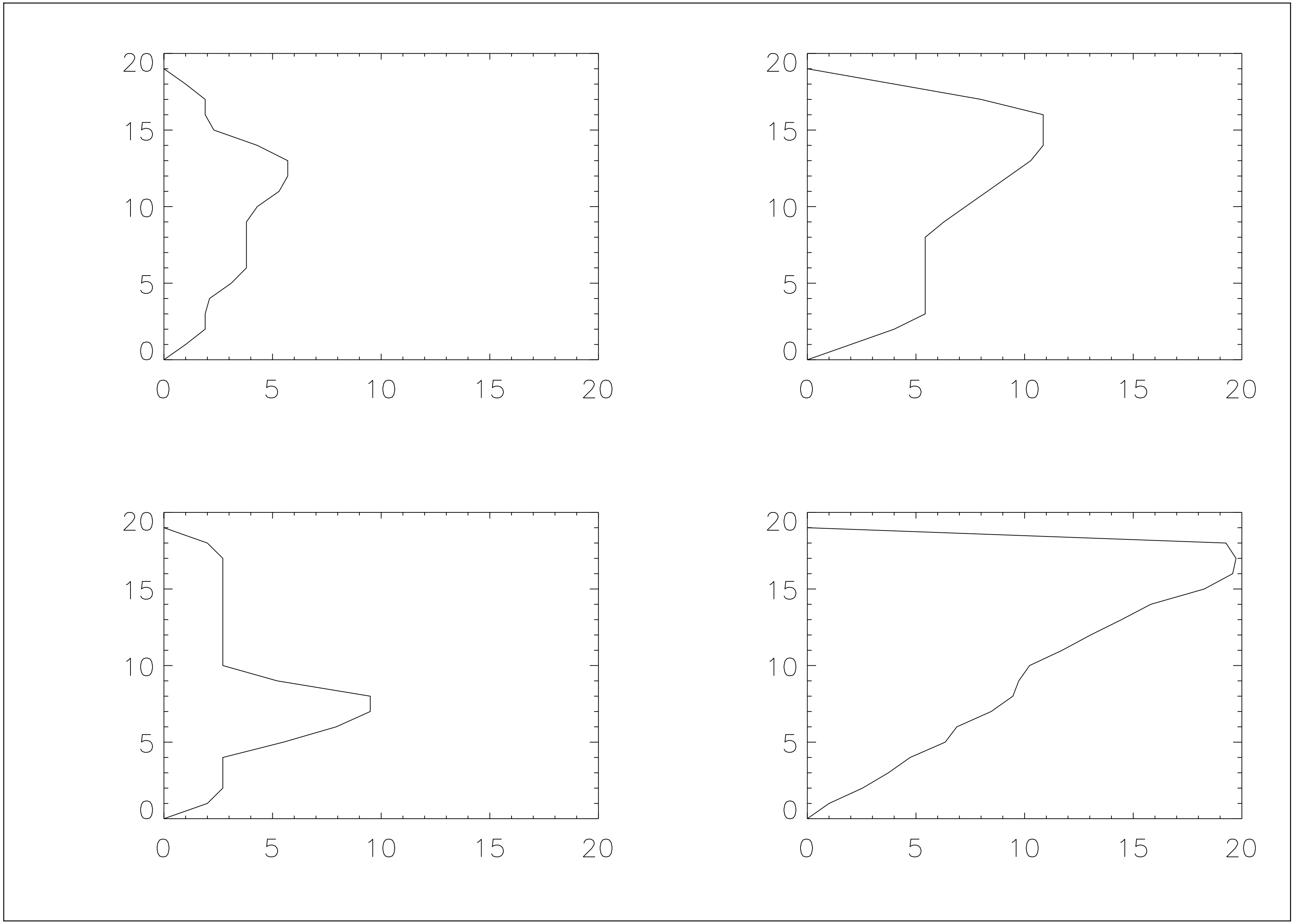
- Start with spherical shape.
- Calculate shape of envelope that is everywhere at the angle of repose.
- If envelope is close to spherical shape, can repeat using envelope as original shape until process converges.
- But envelope is far from original shape, so cannot iterate to solution.



6
4
2
0
−2
−2
0
2
4
6

Irregular Shape Approach

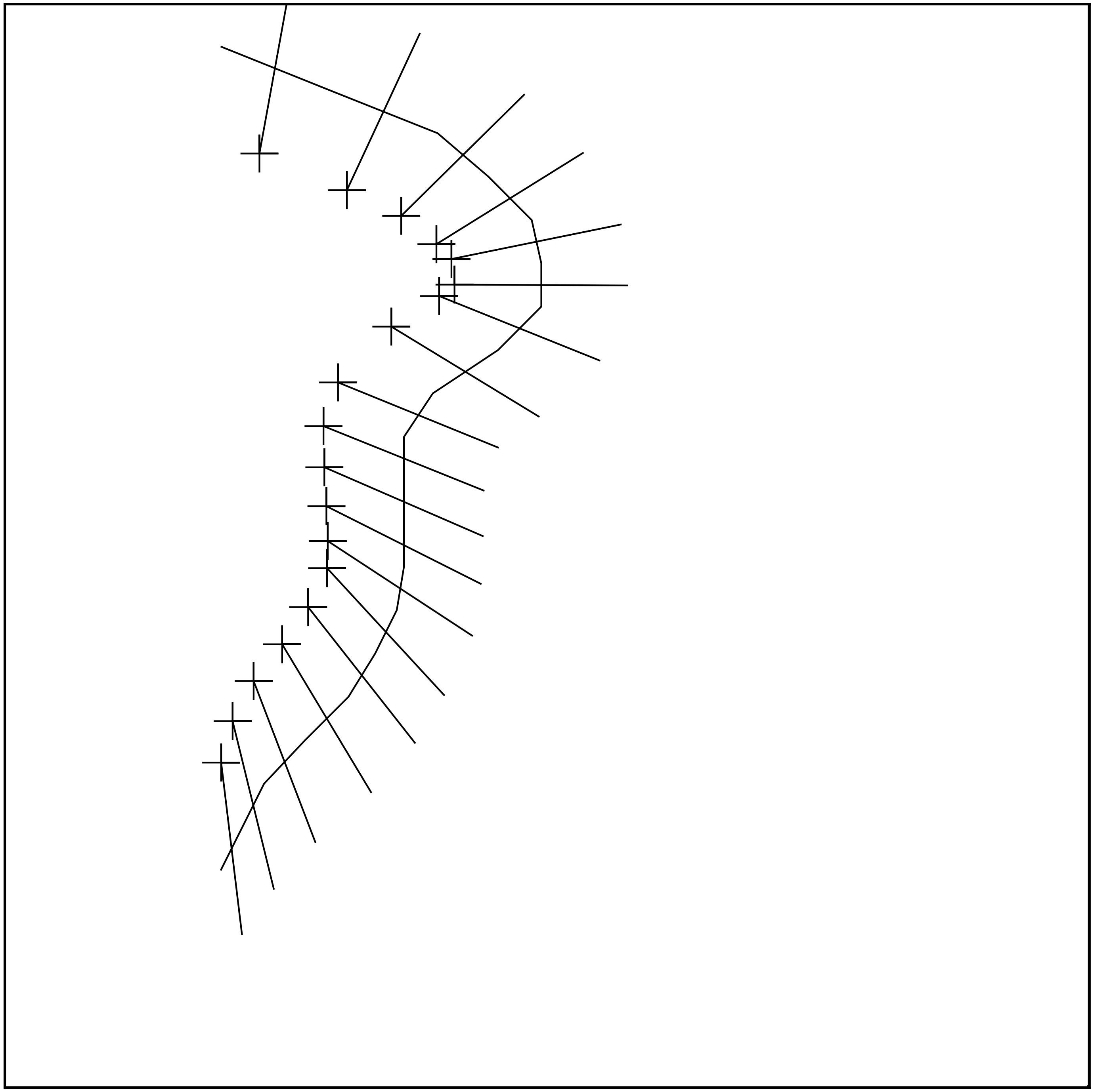
- Use constrained random walk to generate lots of different shapes.
- Fill shape with mass points and calculate g on each surface facet, comparing to the surface normal to find the local slope.
- Investigate slope values.



20
15
10
5
0
0
5
10
15
20

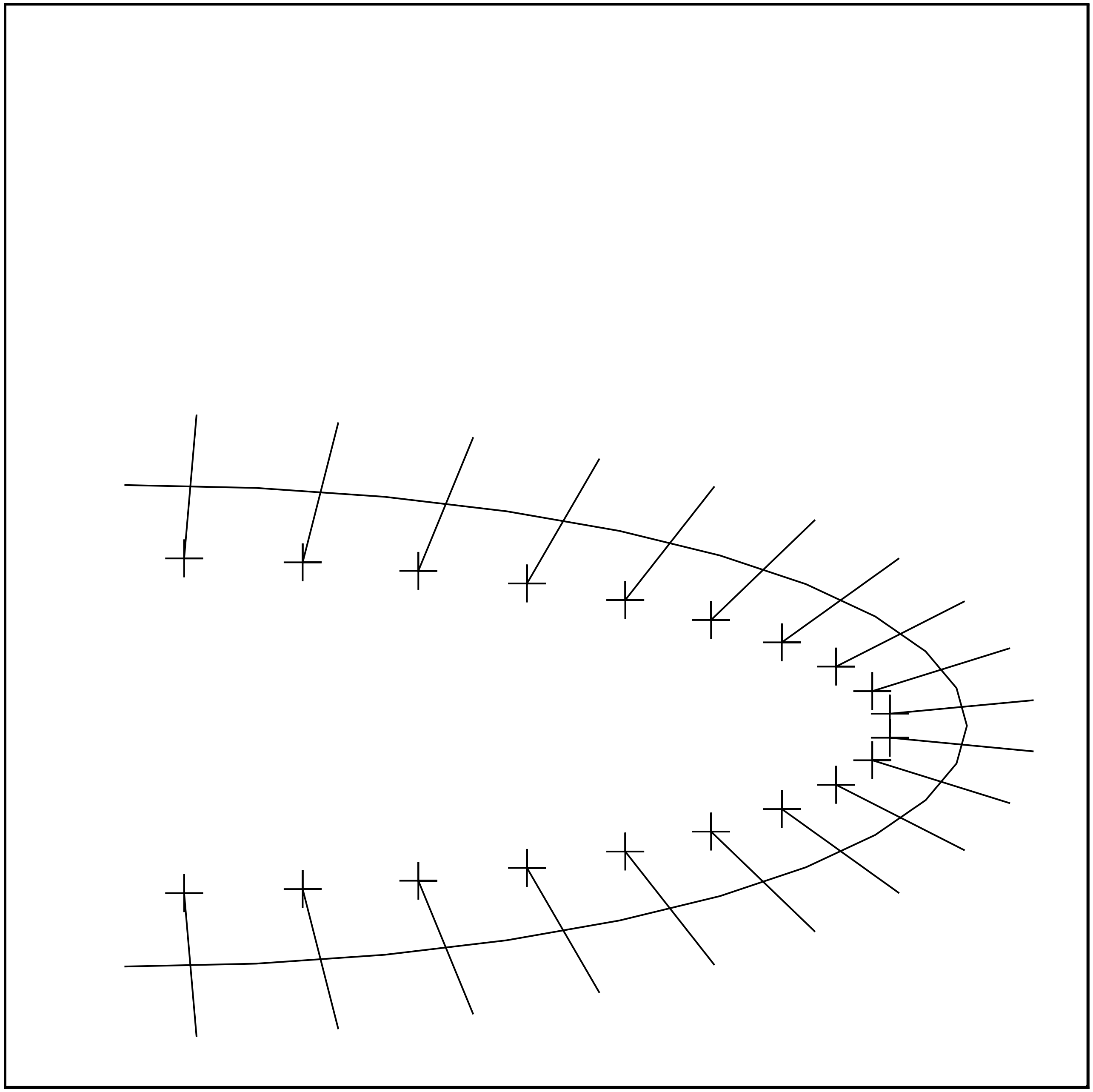
Irregular Shape Results

- Reject 153/200 shapes for having local slopes >30^{o} on >3 of their 19 surface facets.
- Exclude these "scarps" on remaining 47 shapes and calculate mean local slope.
- Only 5 shapes have mean local slope exceeding 15^{o}, none exceeding 18^{o}.
- Not very promising…



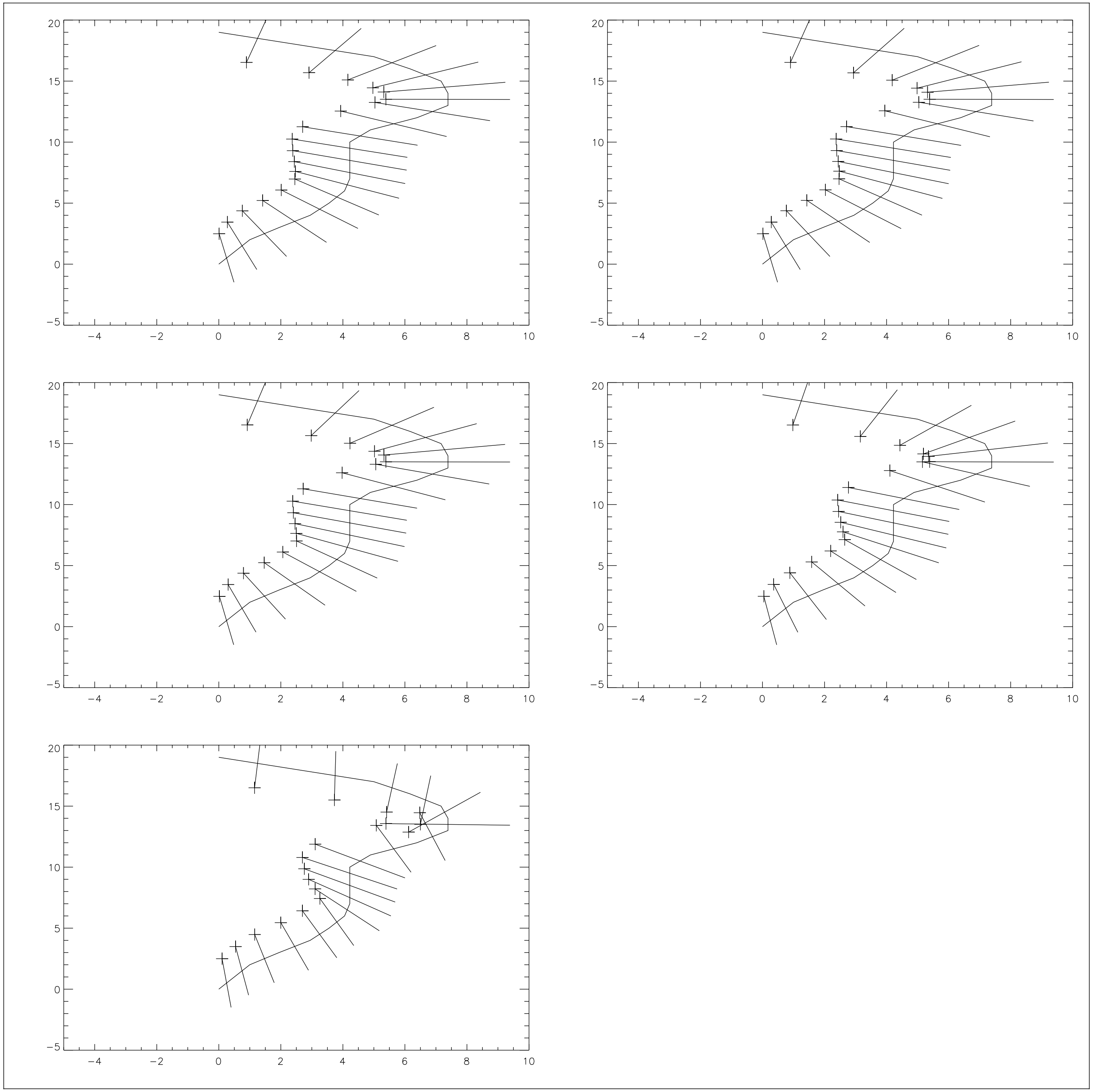
Ellipsoidal Shape Approach

- Both mean and maximum slope are functions solely of axial ratio.
- Find numerically that an axial ratio of ~0.3 gives a maximum slope of $<30^o$ and a mean slope of 19^o.
- Immediately more successful than irregular shape approach.
- Analytical solution should be possible.



Effects of Rotation

- Size independent, controlled by $\omega^2/\rho G$.
- Best ellipsoidal shape always better than best irregular shape.
- No large changes in best mean slope for either class of shapes until rotational effects completely dominant.
- Best irregular shapes tended to remain best over large range of $\omega^2/\rho G$.
- Best ellipsoidal shape changes with $\omega^2/\rho G$, but axial ratio does not change monotonically.


20
15
10
5
0
−5
−4
−2
0
2
4
6
8
10

Future Work

- More irregular shapes and other methods of generating them.
- Analytical study of ellipsoidal shapes.
- “Dripping Sand” approach

Conclusions

- An angle of repose-limited shape is an important end-member for possible asteroid shapes.
- Work to date on finding such a shape is inconclusive, though some ideas for future work are promising.