

Predicting radio occultation uncertainties

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Motivation and Approach

- I want to propose an RS instrument.
 - How do I know what instrumental properties are required to achieve scientific goals?
 - How do I convince reviewers that the instrument requirements I claim are correct?
 - Also useful for understanding old datasets
- Approach
 - Aim for back-of-the-envelope, order-of-magnitude calculation
 - Try to keep physical explanations in mind, not blinded by equations

$$\alpha_j(a_j) = -2a_j \int_{r=r_j}^{r=\infty} \frac{d \ln \mu(r)}{dr} \frac{dr}{\sqrt{(\mu(r)r)^2 - a_j^2}}$$

$$v(r) = v_j \exp\left(\frac{-(r - r_j)}{H}\right)$$

$$\alpha(a) = v(r) \sqrt{\frac{2\pi a}{H}}$$

Usual relationship between bending angle, α , and refractive index, μ

Assume exponential shape for refractivity, v , profile, reasonable for high altitudes

After some algebra and some assumptions that various things are “small”, obtain this equation

Approximate relationship between frequency, f , and bending angle, α , that assumes “small” bending angle

$$\frac{\Delta f}{f} \approx \frac{V\alpha}{c}$$

Relationship between frequency shift, Δf , and refractivity, v .

$$\Delta f(a) \approx \frac{fVv(r)}{c} \sqrt{\frac{2\pi a}{H}}$$

Can now relate instrumental properties to smallest detectable Δf and atmospheric properties to smallest detectable refractivity

Frequency uncertainties

$$s(t) = A(t) \sin(2\pi f(t)t) + \theta(t)$$

Signal s



Ignore variations in amplitude A



Phase noise characterizes variations in f
(Allan deviation)



Thermal noise

Thermal noise

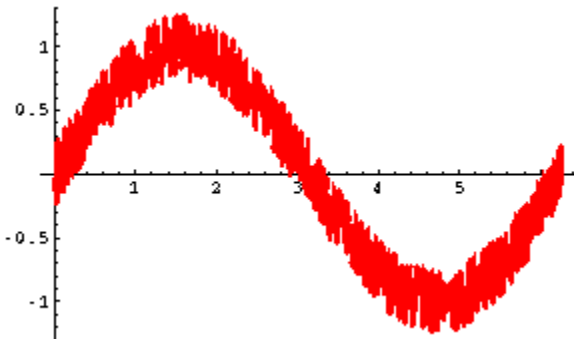
$$\sigma_{thermal} = \frac{\sqrt{2BN_0/C}}{2\pi\tau_{thermal}}$$

$2B$ = Noise bandwidth (Hz)

N_0 = Noise power ($W\ Hz^{-1}$)

C = Signal power (W)

τ -thermal = Time interval (s)



Suppose signal amplitude is $A \sin \phi + \theta$

Need $A \sin \phi > \theta$ to recognize end of one cycle

For small θ/A , uncertainty in phase is therefore θ/A

Hence uncertainty in phase is $\sqrt{2BN_0/C}$

Uncertainty in number of cycles in τ -thermal is

$$\frac{\sqrt{2BN_0/C}}{2\pi}$$

Uncertainty in frequency is $\frac{\sqrt{2BN_0/C}}{2\pi \tau\text{-thermal}}$

Geophysical uncertainties

$$\sigma_V = \frac{c\sqrt{2BN_0/C}}{2\pi f\tau_{\text{thermal}}}$$

Uncertainty in velocity for gravity tracking flyby dominated by thermal noise in one-way mode

$$\sigma_{N_e} \approx \frac{4\pi\sigma_{\Delta f} f c m_e \epsilon_0}{V e^2} \sqrt{\frac{2\pi H_p}{R}}$$

Uncertainty in electron density after inserting expression for ionospheric refractivity

$$\sigma_{n_n} \approx \frac{c\sigma_{\Delta f}}{V f \kappa} \sqrt{\frac{H_n}{2\pi R}}$$

Uncertainty in neutral number density after inserting expression for neutral atmospheric refractivity

Dependence on $\sigma_{\Delta f}$, frequency f , speed V , $(H/R)^{0.5}$, refractive volume k

Nature and NASA define V , $(H/R)^{0.5}$ and k

This leaves f and $\sigma_{\Delta f}$ for the experimenter to influence

Representative parameters and resultant uncertainties in electron and neutral number densities for radio occultations at Venus, Mars, Jupiter and Titan.

	Venus	Mars	Jupiter	Titan
$\sigma_{\Delta f}$ (mHz)	10	10	10	10
f (GHz)	8.4 ^g	8.4 ^k	2.3 ^o	2.3 ^s
V (km s ⁻¹)	7.0 ^h	3.4 ^l	14 ^p	5.6 ^t
H_n (km)	7 ⁱ	10 ^m	25 ^q	20 ^u
H_p (km)	10 ^j	25 ⁿ	1000 ^r	130 ^v
R (km) ^a	6050	3400	70,000	2575
κ (m ³) ^b	1.8×10^{-29}	1.8×10^{-29}	6.2×10^{-30}	1.1×10^{-29}
α (rad) ^c	2.7×10^{-2}	1.8×10^{-4}	3.6×10^{-2}	3.6×10^{-2}
Δf (Hz) ^d	5.4×10^3	1.7×10^1	3.9×10^3	1.5×10^3
σ_{Ne} (m ⁻³) ^e	1.5×10^9	6.3×10^9	5.8×10^8	2.7×10^9
σ_{nn} (m ⁻³) ^f	3.8×10^{19}	1.3×10^{20}	1.1×10^{20}	7.4×10^{20}

← 10 mHz guess based on MGS
Everyone reports an Allan dev., but never thermal noise

Venus (VEX) – Ne uncertainty is predicted well, but neutral uncertainty is 10x smaller than smallest pressure (1 Pa) reported in Tellmann et al. (2009), perhaps due to not reporting pressure at upper boundary with fixed T

Mars (MGS) – Predictions are within 50% of mean observed uncertainties

Jupiter (Voyager) – Predicted Ne uncertainty agrees with Hinson et al. (1998), but predicted neutral uncertainty is 100-1000x smaller than Lindal et al. (1981). Perhaps Lindal used very different bandwidths and integration times than Hinson...?

Titan (Cassini) – Predicted uncertainties are at least 10x too large due to assumed 10 mHz frequency uncertainty being too large, but this work predicts Δf of 1 mHz at $Ne=1E9$ cm⁻³ which agrees with Kliore et al. (2008)